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Review: Brian Clegg, _A Brief History of Infinity: The Quest to Think the Unthinkable_, Robinson, London, UK, 2003

Pages: 255

Past Finding Out

This is an accessible look into a mathematically confounding concept which touches everyone's lives in some way.

Ancient philosophers relevant to the topic are summarised showing that 'new' ideas haven't changed much since Plato's Greek Academy 2,500 years ago (e.g. the existence of objective morality), or Augustine in AD400 and 'multiverse' speculations of his day.

While the maths presented can be understood, understanding infinity would require an infinitely intelligent mind, something only God has:

"Great is our Lord, and of great power: his understanding is infinite."
Psalm 147.5, Authorized Version

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I) To Infinity and Beyond (pp. 1-3)

II) Counting on Your Fingers (pp. 4-)

Zeno was a student of Parmenidies (b. 539BC) who joined a school in Elea, southern Italy. The doctrine taught was of oneness and permanence.

Aesop was a contemporary of Zeno.

Zeno's Arrow asks how one can determine if the arrow is moving or its environment if a snapshot were taken of it).

III) A Different Mathematics (pp. 19-32)

The Greek myriad (M) equalled 10,000.

Infinity was απειρον to the Greeks.

Plato founded a school in Athens in 427BC, in a building belonging to a man named Ακαδεμος which was situated in a grove of trees. It lasted until AD529.

IV) The Power of Number (pp. 33-46)

A googleplex is a one with a googol of noughts after it (i.e $10\verb|^|10^{100}$).

Pythagoras was born c569BC on the Ionian Greek island of Samos to a merchant father. He taught his inner circle students (the μαθεματικοι) that reality was mathematics and that having power over numbers would create power over nature.

V) The Absolute (pp. 47-56)

In AD204, a Roman Plonitus created Neoplatonism which argued that there must be an infinite One, i.e. God, for if He was deficient on someway then there was something else or some other being beyond Him.

In *The City of God*, Augustine said there were some who supposed there are numberless worlds, or one which is born again at fixed intervals without end.

VI) Labelling the Infinite (pp. 57-75)

Leonardo of Pisa (aka "Fibonacci") translated *De numero Indorum*.

Hippasus of Metapontium was supposedly drowned by the Pythagoreans for proving that $2^{1/2}$ was an irrational number, thereby destroying the purity of numbers. The Greeks had no concept of such numbers.

Irrationals are said to lie in the "cloud of infinity".

A code replaces words but a cipher individual letters.

VII) Peeking Under the Carpet (pp. 76-92)

Given a wheel within a wheel, if the longer line traced by the larger travels as much as does the smaller given a single quarter-turn, are there two infinities?

VIII) The Indivisible Mystery (pp. 93-103)

A circle can be sliced into tiny pieces and stacked on top of each other like a quasi-rectangle having a width of the circle's radius, and a height have of its circumference (since this is how the opposite pieces would stack up).

The procedure for calculating the area under a curve was called "Quadrature".

IX) Fluxion Wars (pp. 104-127)

Newton called his calculus the method of fluxions.

The Great Priority Dispute was between Newton and Leibniz for the discovery of calculus.

Leibniz's *d* was equivalent to Newton's *o*.

Atheist mathematicians only believe what they can see, and are blind to the problem of fluxions or differential calculus. The idea of 'tending to' was created to get around the uncomfortable metaphysical consequences of using infinity.

X) Paradoxes of the Infinite (pp. 128-137)

Bolzano was forcefully retired at 39 (some believe due to the Jesuits).

XI) Set in Stone (pp. 139-156)

The nature of infinity is inextricably tied to set theory.

John Venn was born in Hull in 1834.

George Boole invented Boolean algebra.

Academic instruction was traditionally in Latin, meaning a university like Paris could take in students from across Europe. It has only been recently

with the dominance of the US and the internet which made English the dominant tongue.

XII) Thinking the Unthinkable (pp. 157-165)

An infinity containing all countable numbers is $\aleph_0$ and $\aleph_0 + \aleph_0 = \aleph_0$, also $\aleph_0 * \aleph_0 = \aleph_0$, etc.

XIII) Order Versus the Cardinals (pp. 166-171)

The smallest ordinal infinity is ω, after which *transfinite* numbers exist (e.g. $\omega+1$, $\omega+2$, etc.).

Two infinite sets are only equivalent if they have the same element correspondences. For this reason, $2+\omega \neq \omega+2$.

XIV) An Infinity of Infinities (pp. 172-187)

Rene Descartes (from where Cartesian coordinates came from) attended the La Fleche Jesuit school.

Multiplying a number by another raised to the power of zero doesn't do anything which is how the rule $x^0=1$ is derived.

A greater infinity of $\aleph_1 = 2^{\aleph_0}$.

XV) Madness and Sanity (pp. 188-207)

Peer Review relies on the honesty and transparency of those operating the system and it has terrible vulnerability to corruption.

Kurt Godel strictly controlled his diet as he thought others were out to poison him.

Some the Zermelo-Fraenkel Axioms:

-There exists at least one set.

-Two sets are equal if and only if they have the same elements.

-However elements are chosen from one set, the result will always form a set.

-For any two sets there exists a parent set.

-For every collection of multiple sets there exists another which contains all of at least one of the sets.

XVI) <u>Infinitesimally Small</u> (pp. 209-216)

XVII) <u>Infinity to Go</u> (pp. 217-235)

A new concept called "potential infinity" has been proposed.

XVIII) <u>Endless Fascination</u> (pp. 236-243)

Hilbert's Hotel with its infinite rooms, and Gabriel's horn with finite volume of π but infinite surface area.